

# On the extension of a balanced mixed model

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## Abstract

A model with orthogonal block structure, OBS, is a mixed model whose variance-covariance matrix is a linear combinations of known pairwise orthogonal orthogonal projection matrices, that add up to the identity matrix. When the orthogonal projection matrix on the space spanned by the mean vector commutes with the variance-covariance matrix we have a special class of OBS, models with commutative orthogonal block structure, COBS. This commutativity condition of COBS is a necessary and sufficient condition for the least square estimators, LSE, to be best linear unbiased estimators, BLUE, whatever the variance components.

Using the algebraic structure of the models, based on commutative Jordan algebras, and B-matrices, we study the possibility of obtaining COBS from the extension of balanced mixed models.

## Keywords

B-matrices, Jordan Algebra, Mixed models, Models with commutative orthogonal block structure.

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