Estimation of parameters under a generalized growth curve model

Katarzyna Filipiak[1] **and Daniel Klein**[2]

[1] *Poznań University of Technology, Poland*
[2] *P. J. Šafárik University, Košice, Slovakia*

Abstract

Let us consider an experiment, in which p characteristics are observed in q time points for each of n treatments. The data from such an experiment are arranged in three-indices matrix (tensor of order three) and can be modeled using a generalize growth curve model

$$\mathcal{Y} = (\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})\mathcal{X} + \mathcal{E},$$

where $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})\mathcal{X}$ is a product of tensor $\mathcal{X}$ from each of three "sides" by matrices $\boldsymbol{A} \in \mathbb{R}^{n\times n_1}$, $\boldsymbol{B} \in \mathbb{R}^{p\times p_1}$ i $\boldsymbol{C} \in \mathbb{R}^{q\times q_1}$ respectively, i.e.,

$$((\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})\mathcal{X})_{kij} = \sum_{\alpha=1}^{n_1}\sum_{\beta=1}^{p_1}\sum_{\gamma=1}^{q_1} a_{k\alpha} b_{i\beta} c_{j\gamma} x_{\alpha\beta\gamma};$$

cf. Savas and Lim (2008).

Assuming independence of treatments, it is natural to study a doubly-separable variance-covariance matrix of the tensor of observations, which can be presented as a Kronecker product of three matrices, where one of these matrices is identity of order n. The aim of this paper is to determine the maximum likelihood estimators of unknown parameters (expectation and variance-covariance matrix) under a generalized growth curve model.

Presented results are some generalization of the paper by Srivastava et al. (2009).

Keywords

Generalized growth curve model, Maximum likelihood estimates, Block-trace operator, Partial-trace operator.

References

[1] Savas, B. and L.-H. Lim (2008). Best multilinear rank approximation of tensors with quasi-Newton methods on Grassmannians. Linköping University Report, LITH-MAT-R–2008-01–SE.

[2] Srivastava, M., T. von Rosen and D. von Rosen (2009). Estimation and testing in general multivariate linear models with Kronecker product covariance structure. *Sankhyā* 71-A, 137–163.