Kemeny's Constant and an Analogue of Braess' Paradox for Markov Chains

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Abstract

A square matrix that is entrywise nonnegative and has all row sums equal to 1 is called a stochastic matrix, and such matrices play a central role in the study of Markov chains. Given a stochastic matrix A, Kemeny's constant K(A) measures the expected number of steps required for the Markov chain to transition from a given initial state to a randomly chosen final state.

In this talk, we give a brief introduction to Kemeny's constant. We will then explore an analogue of Braess' paradox (wherein adding a road to a network can have the counter-intuitive effect of increasing travel times). Specifically, we will discuss how adding an edge into an undirected graph can increase the value of Kemeny's constant for a certain Markov chain that is naturally associated with the graph.

Keywords Kemeny's constant, stochastic matrix, random walk on a graph.