The deviation matrix and quasi-birth-and-death processes

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Abstract

The deviation matrix is defined as $D = \sum_{n \geq 0}(P^n - \mathbf{1}\,\boldsymbol{\pi}^{\mathrm{t}})$ where P is the transition matrix of an irreducible, positive recurrent Markov chain, and $\boldsymbol{\pi}$ is its stationary probability vector. It is closely related to the equation $(I - P)\boldsymbol{x} = \boldsymbol{r} + w\mathbf{1}$, known as Poisson's equation, where $\boldsymbol{r}$ is a given vector, and it plays an important role in the analysis of Markov chains: one may recall its connections to the sensitivity analysis of the stationary distribution of a Markov chain, and to the Central Limit theorem for Markov chains. If the state space is finite, then the deviation matrix is the group inverse of $I - P$ in discrete time; in continuous time, it is the group inverse of the generator.

As is often the case in Markov chains theory, the deviation matrix may be determined by purely algebraic arguments, or by following a probabilistic approach. I shall focus on quasi-birth-and-death processes (QBDs), that is, Markov chains on a strip in the two-dimensional state space $\mathbb{N}^2$, and I shall show how one may exploit the special transition structure of QBDs, and the physical interpretation of the deviation matrix, in order to obtain an explicit expression in terms of easily obtained quantities.

My presentation is based on joint work with D. Bini, S. Dendievel, Y. Liu and B. Meini [**?**, **?**]

Keywords

Poisson equation, QBD process, group inverse, deviation matrix, matrix difference equation.

References

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